

Formulae Sheet for Economics Part IIA Paper 6

You should be able to recall these formulae in the exam, in whatever form you prefer. I cannot guarantee that the list is exhaustive, but it should cover the great majority of the formulae you will need.

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Calculus

$$\frac{d}{dx} \ln(kx) = \frac{1}{x}$$

$$\frac{d}{dx} e^{kx} = ke^{kx}$$

$$\text{Differentiation of a product: } \frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$\text{Integration by parts: } \int u \frac{dv}{dx} dx = [uv] - \int v \frac{du}{dx} dx$$

$$\text{Quadratic formula: } ax^2 + bx + c = 0 \text{ has solution } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of a convex function: $f((1-t)a + tb) \leq (1-t)f(a) + tf(b)$ for $0 \leq t \leq 1$

$$\text{Total differential: } df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \text{ unless } f \text{ is discontinuous}$$

$$\text{Implicit function theorem: } \frac{dy}{dx} = - \frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$

Homogeneous function $f(x_1, x_2, \dots, x_n)$, of degree m obeys:

$$f(tx_1, tx_2, \dots, tx_n) = t^m f(x_1, x_2, \dots, x_n) \text{ and (Euler) } x_1 \frac{\partial f}{\partial x_1} + x_2 \frac{\partial f}{\partial x_2} + \dots + x_n \frac{\partial f}{\partial x_n} = mf$$

$$\text{Binomial series: } (1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \dots$$

$$\text{or: } (a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}b^2}{2!} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + \dots + nab^{n-1} + b^n$$

$$\text{Maclaurin series: } f(x) = f(0) + xf'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \dots$$

$$\text{Taylor series in 1D: } f(a+x) = f(a) + xf'(a) + \frac{x^2 f''(a)}{2!} + \frac{x^3 f'''(a)}{3!} + \dots$$

$$\text{exponential series: } e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\text{log series: } \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots$$

Taylor series in 2D:

$$f(a+x, b+y) = f(a, b) + \left(x \frac{\partial f}{\partial x} \Big|_{(a,b)} + y \frac{\partial f}{\partial y} \Big|_{(a,b)} \right) + \frac{1}{2!} \left(x^2 \frac{\partial^2 f}{\partial x^2} \Big|_{(a,b)} + 2xy \frac{\partial^2 f}{\partial x \partial y} \Big|_{(x,y)} + y^2 \frac{\partial^2 f}{\partial y^2} \Big|_{(a,b)} \right) + \dots$$

ie:

$$f(a+x, b+y) = f(a, b) + (x, y) \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix} \Big|_{(a,b)} + \frac{1}{2!} (x, y) \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix} \Big|_{(a,b)} \begin{pmatrix} x \\ y \end{pmatrix} + \dots$$

$$\text{l'Hôpital: } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} \text{ when } \lim_{x \rightarrow a} f(x) = 0 \text{ and } \lim_{x \rightarrow a} g(x) = 0$$

$$\lim_{n \rightarrow \infty} (1 + x/n)^n = e^x$$

$$\text{Geometric progression: } \sum_{i=0}^{\infty} ar^i = \frac{a}{1-r}$$

$$\text{\& you should either know, or be able to work out from this, that } \sum_{i=0}^n ar^i = \frac{a(1-r^{n+1})}{1-r}$$

In whatever form you choose to remember them: Mean Value Theorem, Envelope Theorem, Shephard's Lemma, Roy's Identity.

Statistics

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

Bayes: $P(A | B) = \frac{P(B | A)P(A)}{P(B)}$

$P(A \cap B) = P(A)P(B)$ if A and B are independent

$P(A \cap B) + P(A \cap \bar{B}) = P(A)$

Binomial probability of r successes n trials = ${}^n C_r p^r (1-p)^{n-r}$ where p = probability of success in any one trial.

Poisson probability of r successes = $\frac{\lambda^r e^{-\lambda}}{r!}$

Normal distribution is $N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Beta distribution $Beta(\alpha, \beta) = Kx^{\alpha-1}(1-x)^{\beta-1}$ (K=normalising constant) for $0 \leq x \leq 1$

Conditional density function: $g(y | x) = \frac{f(x, y)}{h(x)}$ where f(x,y) = joint density function

Marginal density function: $h(x) = \int f(x, y)dy$

Continuous form of Bayes' Theorem: $f(x | data) = \frac{prob(data | x)f(x)}{\int prob(data | x)f(x)dx}$

Uncentred and centred r^{th} moments: $\mu_r' = E(x^r)$ and $\mu_r = E((x - \mu)^r)$

$E(x^r) = \sum_x x^r Pr(x)$ (discrete) or $E(x^r) = \int x^r f(x)dx$ (continuous)

$E(X_1 + X_2) = E(X_1) + E(X_2)$

$E(X_1 + k) = E(X_1) + k$

$E(kX_1) = kE(X_1)$

$E(X_1 X_2) = E(X_1)E(X_2)$ if X_1 and X_2 are independent

$var(X) = E[(X - E(X))^2] = E(X^2) - [E(X)]^2$

$var(X_1 + k) = var(X_1)$

$$\text{var}(kX_1) = k^2 \text{var}(X_1)$$

$$\text{var}(X_1 + X_2) = \text{var}(X_1) + \text{var}(X_2) \text{ if } X_1 \text{ and } X_2 \text{ are independent}$$

$$\text{Moment generating function: } M_X(t) = E(e^{tx}) \text{ and } \left. \frac{d^r M_X(t)}{dt^r} \right|_{t=0} = \mu_r'$$

$$M_{X_1+X_2+X_3+\dots}(t) = M_{X_1}(t)M_{X_2}(t)M_{X_3}(t)\dots$$

$$M_{X+a}(t) = e^{at}M_X(t)$$

$$M_{bX}(t) = M_X(bt)$$

$$\text{Moment generating function of } N(0,1) \text{ is } e^{-t^2/2}$$

$$\text{Cramer-Rao lower bound: } \text{var}(\hat{\theta}) \geq \left[-E\left(\frac{d^2 \log(L)}{d\theta^2}\right) \right]^{-1}$$

$$\text{Chebyshev: } \Pr(|X - \mu| \geq \varepsilon) \leq \frac{\sigma^2}{\varepsilon^2}$$

Linear Algebra

$$A + (B + C) = (A + B) + C$$

$$A(B + C) = AB + AC$$

$$(A + B)^T = A^T + B^T$$

$$AB \neq BA, \text{ in general}$$

$$A^{-1}A = AA^{-1} = I$$

$$A(BC) = (AB)C$$

$$(AB)^T = B^T A^T$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$\det(A^T) = \det(A)$$

$$\det(AB) = \det(A)\det(B)$$

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

Rules of determinants:

Can subtract a multiple of any row from any other row without affecting value

Can expand along any row

Swapping two rows reverses the sign of the determinant

Multiplying all terms in a single row by λ multiplies the determinant by λ

Any rule that applies to rows also applies to columns

Formula for the inverse of a matrix, using cofactors: $A^{-1} = \frac{\text{adj}(A)}{\det(A)}$ and hence A^{-1}

exists if and only if $\det(A) \neq 0$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Cramer's Rule (in whatever form you wish to remember it)

Linear independence: $\alpha_1 \underline{x}_1 + \alpha_2 \underline{x}_2 + \dots + \alpha_n \underline{x}_n = 0$ if and only if $\alpha_i = 0$ for all i

Eigenvalues and eigenvectors: $A\underline{v} = r\underline{v}$ and $\det(A - rI) = 0$

Sum of eigenvalues of A = Trace of A

Product of eigenvalues of A = Determinant of A

Diagonal form $D = P^{-1}AP$ exists if P^{-1} exists, ie if the eigenvalues of A are *distinct*.

The two key rules of vector spaces: if \underline{u} and \underline{v} lie in the space, then $\underline{u} + \underline{v}$ and $r\underline{u}$ must also lie in the space.

Fundamental Theorem of Linear Algebra

$\text{Dim}(\text{Null}(A)) = \text{No. of columns in } A - \text{Rank}(A)$