2011

A1. –

2. (a) Eg $(3 \quad -10 \quad 0)^T$ and $(0 \quad 0 \quad 1)^T$, no; (b) 17.

3. (a) $\begin{pmatrix} 3 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 1 \end{pmatrix}$; (b) No, as the other eigenvalues are 5 and -1.

4. (b)(ii) $\frac{-2}{1+ln3}$.

5. $A = C = 0, B = 2$.

6. (a) $k = \left(k_0 - \frac{\beta}{\alpha}\right) e^{\frac{\alpha t^2}{2}} + \frac{\beta}{\alpha}$, steady state is $k = \frac{\beta}{\alpha}$.

7. (NB: you need a Taylor Series around $\bar{y}$, not zero!) $E(U) = 2^{\bar{y}} + 3^{\bar{y}}$, $var(U) = (2^{\bar{y}} ln2 + 3^{\bar{y}} ln3)^2 \sigma^2 / n$.

8. (a) $\binom{S}{y} \theta^y (1-\theta)^{S-y}$; (b) $K\theta^2(1-\theta)^8$ where K=normalising constant, MLE=0.2; (c) $K\theta^{10}(1-\theta)^{30}$.

9. (a) $cn^{-1}, n^{-1} + n^{-2}$; (b) $c < 1 + n^{-1}$.

B10. (a) one unique solution if $t \neq -8$, no solutions if $t = -8, s \neq -16$, infinitely many solutions if $t = -8, s = -16$; (b) (ii) f^1, f^4; (iii) f^1: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, f^4: $\begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \end{pmatrix}$.

11. (a) (i) $\begin{pmatrix} 1-b & a \\ m & -h \end{pmatrix} \begin{pmatrix} Y \\ r \end{pmatrix} = \begin{pmatrix} I^0 + G \\ M^S - M^0 \end{pmatrix}$, unique solution requires $h(1-b) + am \neq 0$; (ii) $Y = \frac{h(I^0+G)+a(M^S-M^0)}{h(1-b)+am}, r = \frac{m(I^0+G)-(1-b)(M^S-M^0)}{h(1-b)+am}$; (iii) $\Delta Y = \frac{h\Delta G}{h(1-b)+am}, \Delta r = \frac{m\Delta G}{h(1-b)+am}$.

12. (a) $L_1 = \frac{L_0}{\left(\frac{p_1 a_1}{p_2 a_2}\right)^{\frac{1}{b-1}}+1}, L_2 = \frac{L_0}{\left(\frac{p_2 a_2}{p_1 a_1}\right)^{\frac{1}{b-1}}+1}$; marginal increase in national output when L_0 increases is $\lambda = \frac{p_2 a_2 b L_0^{b-1}}{\left(\left(\frac{p_2 a_2}{p_1 a_1}\right)^{\frac{1}{b-1}}+1\right)^{b-1}}$; (c) $\frac{dY}{dp_1} = a_1 L_1^b$.

13. (b) (i) (Take "with support (0,4]" to mean $0 < w \leq 4$): $K = \frac{1}{4}$; (ii) 1; (iii) $\frac{4}{3}$; (c) $\frac{1}{4}$.

14. (b) (i) $n = \frac{\sigma_X^2}{\delta \varepsilon^2}$; (ii) $n = 10^5$; (c) $Prob\left(\left|N(0, \frac{\sigma_X^2}{n})\right| \leq \varepsilon\right)$; (d) Via Normal approx: $n > 96$, via Chebyshev: $n \geq 500$.

15. (a) $(1-\theta)y + \theta y^2, \frac{1}{2} + \frac{\theta}{6}$; (b) $\hat{\theta} = 6\bar{y} - 3$; (c) $\sum_i \frac{-1+2y_i}{(1-\theta)+2\theta y_i} = 0$.