

2010

Paper I

A1.  $x = 1, \frac{3}{2}$  or 2. 2. (a)  $-2xe^{-x^2}$ ; (b)  $2 \sinh x \cosh x$ . 3. (a)  $x \ln x - x$ ; (b) zero.

4. (a) 1. 5. Radius 3 centre (3,1). 6.  $x = e^2, k = e^{-2}$ . 7. (a) 1000; (b)  $\frac{(n+1)(n+2)}{2}$ . 8. -

9.  $x = -2, x = -4$  (if one allows the negative root). 10  $x = \frac{\pi}{8}$  or  $\frac{5\pi}{8}$ .

B11. (a) 2.0305; (b)(i)  $x + x^2 + \frac{x^3}{3}$ ; (ii)  $x + x^2 - \frac{2x^3}{3}$ .

12. (a)  $(1-p)^F$ ; (b)  $\binom{F}{n} p^n (1-p)^{F-n}$ ; (c)  $Fp, Fp(1-p)$ ; (d)  $\binom{F-1}{n} p^n (1-p)^{F-n}$ ;  
(e)  $\sum_{k=0}^{F-n-1} \binom{n-1+k}{k} p^n (1-p)^k$ .

13. (a)  $1, 3 \pm \sqrt{15}$ , eigenvector for  $\lambda = 1$  is  $(1 \ 1 \ 3)^T$  (lost the will to live beyond that);

(b)  $\left(-\frac{3}{2} \ \frac{7}{2} \ -\frac{3}{2}\right)^T$ .

14. (a)(i)  $\frac{12}{13}, \frac{5}{13}$ ; (ii)  $0, \frac{\pi}{6} + 2n\pi$ ; (iii)  $0, -1$ ; (iv)  $-\frac{1}{2}, -\frac{\sqrt{3}}{2}$ ; (b)  $2^{\frac{1}{4}} e^{i\left(\frac{\pi}{12} + \frac{n\pi}{2}\right)}, n = 0, 1, 2, 3$ .

15. (a)  $\mathbf{x} = \frac{\mathbf{a} - (\mathbf{b} \cdot \mathbf{a})}{1 + |\mathbf{b}|^2} (?)$ ; (b)  $\mathbf{x} = \mathbf{a} + \left(\frac{\mathbf{a} \cdot \mathbf{b}}{1 - \mathbf{b} \cdot \mathbf{c}}\right) \mathbf{c}$ , if  $\mathbf{b} \cdot \mathbf{c} = 1$  then  $\mathbf{x} = \mathbf{a} + \gamma \mathbf{c}$  for any  $\gamma$ .

16. -.

17. (a)  $y = -\frac{1}{x} \ln(1 + e^{-1} - x)$ ; (b)  $y = (4(1 + x^3))^{\frac{1}{3}}$ ;

(c)  $y = 2e^{x-1} - 1, y = 4e^{x-1} - \left(\frac{x^2}{2} + x + \frac{3}{2}\right)$ .

18. (a)(i)  $at^{a-1}y + t^{2a-1}b^aaxy'$ ; (ii)  $t^{4a}b^{3a}y'''$ ; (b)  $a = -\frac{1}{3}, b = 3$ .

19. (b)  $2e\delta x + e\delta y$ ; (c)  $u = A \sin 3x \sin 4y + B \sin 4x \sin 3y$ .

20. (a) (i) Yes ( $e^2$ ); (ii) No; (b)  $2 < x < 4$ ; (c) (i) 4; (ii)  $\frac{1}{9}$ .

Paper II

A1.  $\theta = 0.906$  rad. 2. 2 3.  $(\pm 1, 0)$ . 4.  $\pm \frac{\sqrt{28}}{3}$ . 5.  $-\frac{5}{41}, \frac{37}{41}$ . 6.  $\frac{1}{2} - \frac{\sqrt{3}\left(x - \frac{\pi}{3}\right)}{2} - \frac{\left(x - \frac{\pi}{3}\right)^2}{4}$ . 7.  $y = -2x + 1$ .

8.  $\frac{11}{36}$  9.  $y = 1$ . 10.  $y^2z^2 + 4y^3 + x^3y^2$ .

B11. (a)  $x = r \cos \theta, y = r \sin \theta$ ; (c)  $AC^{-2}, C^{-1}$ .

12. (b)  $x = X_0 \cos \sqrt{\frac{k}{m}}t, y = V_0 \sqrt{\frac{m}{k}} \sin \sqrt{\frac{k}{m}}t$ .

13. (a) minimum at (0,0); (b) Saddles at  $(\pm 4, 0)$ , Minimum at (0,2), Maximum at (0, -2).

14. (a) (i) 1.

15.  $\frac{\sin \mu\pi}{\mu\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{2\mu(-1)^n \sin \mu\pi}{\mu^2 - n^2} \cos nx; A = 3, B = 3$ .

16. (0, 2, 2), (0, 0, 0);  $F_1$ : (a) 1, (b)  $\frac{\pi}{2}$ ;  $F_2$ : (a) 0, (b) 0.

17. (a) (i)  $\frac{16}{35}$ ; (ii)  $\frac{7}{16}$ ; (b)  $\frac{\binom{34}{3}\binom{64}{4}}{\binom{100}{7}}$ ; (c) (i)  $\alpha = P(B|A)P(A), \beta = (P(B|A) - 1)P(A) + 1$ ;

(ii)  $\alpha = \text{Min}(P(B|A), P(B|\bar{A})), \beta = \text{Max}(P(B|A), P(B|\bar{A}))$ .

18. (a) 224; (b)  $\frac{1}{2}(e - 1)$ ; (c)  $\frac{81\pi}{4}$

19. (a)  $C = 6$ ; (b)  $D = -1, E = 5$ .

20. (a)  $H_0 = 1, H_1 = 2x, H_2 = 2 - 4x^2$ .

2011

Paper I

- A1. 1. 2. (a)  $-\frac{1}{2}\ln(3-2x) + c$ . 3. (a) (0,1); (b)  $(e, e^{-e})$ .
4. (a) spiral, starting from  $(0,4\pi)$  and going clockwise into origin; (b)  $7\pi/2$ . 5.  $\pi: 3\sqrt{3}$ .
6.  $\pm\frac{\pi}{6}, \pm\frac{5\pi}{6}$ . 7.  $-\ln 4$ . 8.  $e^{-x^2} \sin x$ . 9.  $\frac{1}{x \ln x}$ . 10. (a)  $10^6$ ; (b)  $\frac{8}{7}$ .
- B11. (b)(i)  $\frac{1}{3} - \frac{x^2}{54}$ ; (ii)  $3 \ln 2 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{24}$ ; (iii)  $1 + x + \frac{1}{2}x^2$ .
12. (a)  $\frac{1}{5}e^{-\frac{s}{5}}$ ; (d)  $\frac{\bar{t}}{5+\bar{t}}$ ; (f)  $(1 - \frac{\bar{t}}{5+\bar{t}})e^{\frac{r_0}{5}}$ ; (h)  $\bar{s}$ .
13. (a) (i) 16; (ii)  $\frac{1}{16} \begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}$ ; (iii) 2 and 8;  $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ; (b)  $b = 5, c = -6, d = 5$
- (c)  $2x'^2 + 8y'^2 = 1, B = \begin{pmatrix} 1 & 1 \\ 2 & -2 \end{pmatrix}$ .
14. (d)  $a_m = b_m = 0$  if  $m \neq 1, a_1 = -\sqrt{3}, b_1 = 1$ ; (e)  $2 \cos(\frac{\pi}{3} + x)$ ; (g)  $4\pi$ .
15. (d)  $s = \frac{[a,b,n]}{[b,m,n]}, t = \frac{[a,b,m]}{[a,m,n]}$ .
16. (a)  $y = 2 \ln x - 1$ ; (b)  $y = (x^2 - 1)e^{-x^2}$ ; (c)  $y = \frac{8}{3} \cos 2x - \frac{\sqrt{2}}{3} \sin 2x + \frac{1}{3} \cos x + \frac{1}{3} \sin x$ .
17. (b) (i)  $2, -\frac{\pi}{3}$ ; (ii)  $\sqrt{5}, \tan^{-1} 2$ ; (iii)  $\sqrt{2}, \frac{5\pi}{12}$ ; (b)  $e^{\pm i\pi(\frac{2}{9} + \frac{2n}{3})}$ .
18. (b) Neither (?) (c)  $\mu = 1/\sin x, y = -\frac{1}{x} \ln(\sin x) + \frac{c}{x}$ ; (d)  $(\frac{\partial a}{\partial d})_e$ .
19. (a) zero; (b)  $\pi$  (or  $-\pi$ , if you take  $d\mathbf{S}$  to be in negative y direction); (c)  $4\pi kr^3$ ; (d)  $4\pi$ .
20. (b)  $x^2 + \frac{1}{6x^8} + O(\frac{1}{x^{18}})$ .

Paper II

- A1.  $c = 6$ . 2.  $\ln(x^2 + 3x - 2) + c$ . 3.  $x = -1, -5$ , discontinuities at  $x = -2, 1$ . 4. -.
5.  $Re = \frac{14}{25}, Im = -\frac{23}{25}$ . 6.  $1 + \frac{(x-1)}{2} - \frac{(x-1)^2}{8}$ . 7.  $y = -\ln(1 - \frac{1}{2}\ln(x^2 + 1))$ .
8. Circle, radius 2, centre (0,1). 9.  $y = 12x - 15$ . 10. Zero.
- B11. (a) (i)  $\pi$ ; (ii) Zero; (iii) Zero; (b) (i) and (ii) are both  $\pm 10e\pi\mathbf{k}$  (don't know which until the rim traversal direction is specified?)
12. -.
13. (b)  $-\sqrt{3}$ .
14. (a) (i) 27; (iii)  $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ , anything satisfying  $x + 2y - 3z = 0$ , eg  $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ .
15. (a)  $x = \frac{V_{0x}}{\gamma}(1 - e^{-\gamma t}), z = (\frac{V_{0z}}{\gamma} + \frac{g}{\gamma^2})(1 - e^{-\gamma t}) - \frac{gt}{\gamma}$ ;
- (b)  $z = \frac{g}{\gamma^2} \left[ \ln\left(1 - \frac{\gamma x}{V_{0x}}\right) + \left(\frac{\gamma^2 V_{0z}}{g} + \gamma\right) \frac{x}{V_{0x}} \right]$ .
16. (a)  $E(\frac{2}{3}, \frac{\pi}{6})$ ; (c)  $\frac{\pi}{2a}(3a^2 + b^2)$ .
17. (a)  $\frac{(32!)^3}{64!(16!)^4}$ ; (b)  $l = \frac{3}{4}, p = \frac{2}{7}$ .
18. (a)  $\frac{R^3}{6}$ ; (b)  $8\pi$ .
19. (c)  $\Psi(x, y) = \frac{2V}{\pi} \tan^{-1}\left(\frac{2y}{1-x^2-y^2}\right)$ .
20. (a)  $L_{10} = \frac{1}{10!}x^{10} - \frac{10}{9!}x^9 + \dots$ .