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Value and probability

Abstract: *This paper expresses the Post Keynesian critique of 'fundamental value' and the efficient markets hypothesis using the symbols of Keynes's Treatise on Probability. A distinction is drawn between* ex ante *and* ex post *fundamental value, which coincide in the case of fixed annuities but not for financial assets in general, except in an ergodic system. Keynes's symbols allow expression of the general form of the probability relation represented by* ex ante *fundamental value, highlighting its dependence on unreliable propositions about future events; and also of the conventional basis of valuation which it is only rational to adopt in such circumstances.*

Keywords*: asset pricing, efficient markets, Keynes, expectation, uncertainty*

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The concept of 'fundamental value' is central both to the somewhat battered efficient markets hypothesis and as a reference point for currently modish behavioural finance theory[1], despite Keynes's unanswered critique in *The General Theory* (1936, hereafter *GT*). This paper expresses in the symbols of Keynes's *Treatise on Probability* (1921) the substantive content of section II and the first paragraph of section IV of *GT* Chapter 12 on "The state of long-term expectation" (*GT* 148-149, 152), with a view to clarifying precisely the nature of the flaw in the concept of fundamental value in the real world. As Post Keynesian writers have long emphasised, the Classical concepts of fundamental value and probability are based on an ergodic axiom (Davidson, 1996, 2005); the contribution of this paper is to demonstrate precisely how they are special cases of more general theories that dispense with this counter-factual restriction.

Ex ante and *ex post* fundamental value

Let us begin by distinguishing the market price of an investment or financial claim q_t from its fundamental value in prospect (*ex ante*) q_t^* and its fundamental value in retrospect (*ex post*) q_t^{**}; so that q_t^* is an expectation of the outcome q_t^{**}. Note that both the market price q_t and the *ex post* fundamental value q_t^{**} are observable. It is in the nature of financial assets that they are traded on well-organised markets with well-defined competitive prices, so that q_t can easily be observed. *Ex post* fundamental value q_t^{**} can also in principle be observed, although it is a subject for accountants, and even then, only for those with a peculiarly academic and historical bent. For it is in principle possible, if of little or no commercial importance, to identify the market interest rates and the money yield of an asset over the course of its economic life (*GT* 136), and so the price q_t^{**} that would have warranted the holding of the asset at any

[1] For recent introductions to the literature see Malkiel (2003) and Shiller (2003). See also Fama (1970) and for a Post Keynesian critique, Glickman (1994).

time as an alternative to a debt, given perfect foresight[2]. By contrast, the *ex ante* fundamental value q_t^* is intrinsically unobservable, except in the case of fixed annuities, and we shall find in due course that this unobservability presents an insuperable problem.

We set out from common ground, the case in which $q_t = q_t^* = q_t^{**}$. The market price in equilibrium of a 'gilt-edged' claim to a series of fixed future money receipts (what we shall call a 'fixed annuity' - Keynes uses the term slightly differently, *GT* 135) is the net present value of the series, which can be expressed as:

$$q_t = q_t^* = q_t^{**} = \sum_1^N d_{t+i} \frac{1}{\left(1 + R_{t+i}\right)} \qquad (1)$$

where N is the number of discrete time periods over which the series extends, d_{t+i} is the receipt due at time $t+i$, and R_{t+i} is the interest on a loan of a unit of money at time t for i periods. The three q's, with and without asterisks, are equivalent because both d_{t+i} and R_{t+i} are known at any time, given a market for fixed-rate debts of comparable maturities. Equation (1) can be simplified by the assumptions that the stream of future receipts is a perpetual annuity growing in each period at a constant rate g (something like UK 'consols' but with an escalating coupon) and that the rate of interest in each period is a constant r, to give:

$$q_t = \frac{d_{t+1}}{\left(r - g\right)}, \text{ such that } \left(r > g\right) \qquad (2)$$

which looks very much like the standard dividend discount model for the valuation of equity securities. However it is a considerable leap from the equilibrium price of fixed annuities to the market prices of financial assets in general, and the various assumptions required to make such a leap represent the heart of the controversy.

If, as an alternative to holding the claim to maturity, an investor can transfer the claim at an earlier date (including the next period), the relation between the present and future market prices q_t and q_{t+1} is, in equilibrium, given by the 'no arbitrage opportunity' condition:

$$q_t = \frac{d_{t+1} + q_{t+1}}{\left(1 + R_{t+1}\right)} \qquad (3)$$

where today's asset price equals the net present value of the sum of tomorrow's dividend and tomorrow's asset price. It is a small but significant step from equation (3) to the 'rational expectations hypothesis' and the claim that rational, well-informed agents do not make systematic errors in forming their expectations. This is expressed by incorporating into (3) an 'expected value' operator:

$$q_t = \mathrm{E}_t\left[\frac{d_{t+1} + q_{t+1}}{\left(1 + R_{t+1}\right)}\right] \qquad (4)$$

[2] No meaningful definition of 'risk adjusted' *ex post* fundamental value is possible, since the original risk cannot in general be measured with hindsight from outcomes, as we shall see.

and by substitution and the use of the 'law of iterated expectations', that $E_t[E_{t+1}[q_{t+2}]] = E_t[q_{t+2}]$, one solution[3] of equation (4) in q_t looks very like equation (1) with the addition of the expected value operator:

$$q_t = q_t^* = E_t\left[\sum_{1}^{N} d_{t+i} \frac{1}{(1+R_{t+i})}\right] = E_t\left[q_t^{**}\right] \qquad (5)$$

Equation (5) states that q_t^* is the expected value of the prospective yield, in turn assumed to be a stochastic variable with a random disturbance term. This crucial assumption takes the only source of uncertainty to be the disturbance term, of which the expected value is zero, so that equations (1) and (5) are otherwise equivalent. If the disturbance term is normally distributed, uncertainty becomes synonymous with variance or 'volatility'. Equations (1) and (5) are indeed equivalent in the case of a fixed annuity, where $E_t[d_{t+i}] \equiv d_{t+i}$ and $q_t^* = q_t^{**}$.

In the absence of a bubble, equation (5) represents the efficient markets hypothesis (EMH) that $q_t = q_t^*$, i.e. observed prices in competitive financial markets represent *ex ante* fundamental values[4]. Behavioural finance (Shleifer, 2000) and complexity theory (Sornette, 2003) identify investor psychology and the limits of arbitrage as systematic sources of divergence from fundamental value, but the critique now offered here, following Keynes, raises the prior question whether market prices of assets other than fixed annuities can ever represent an accurate expectation of fundamental value.

In a theory of competitive equilibrium, the prospective yield of a capital-good represents a set of expected equilibrium prices and outputs reflecting supply and demand at future dates. To sustain the EMH requires one of two assumptions, either

EMH-A — *the world behaves as if complete futures and insurance markets extend to the horizon of long-term expectation; or*

EMH-B — *a process of trial and error leads to a convergence of expectations on their equilibrium values.*

EMH-B implies EMH-A; while EMH-A is sufficient on its own, if no more than an assertion, given the absence of the required markets. It is an understanding that the world is such as to make both these assumptions invalid that leads Keynes to write

> "Or, perhaps, we might make our line of division between the theory of stationary equilibrium and the theory of shifting equilibrium - meaning by the latter the theory of a system in which changing views about the future are capable of influencing the present situation. *For the importance of money essentially flows from its being a link between the present and the future.* We can consider what distribution of resources between different uses will be consistent with equilibrium under the influence of normal economic motives

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[3] Equation (5) is only one solution of the the first-order difference equation (3); the general solution may include a deterministic or stochastic 'bubble term'. See Camerer (1989).

[4] The reference here is to the 'strong' EMH (where prices reflect all information available). The 'semi-strong' EMH (where prices reflect all information available to the market) allows for asymmetric information between insiders and outsiders, and the possibility of insider trading profits; while the 'weak' EMH holds only that prices already reflect the information embodied in past prices, leading to a random walk in prices, since new information ('news') is unpredictable.

> in a world in which our views concerning the future are fixed and reliable in all respects; - with a further division, perhaps, between an economy which is unchanging and one subject to change, but where all things are foreseen from the beginning. Or we can pass from this simplified propaedeutic to the problems of the real world in which our previous expectations are liable to disappointment and expectations concerning the future affect what we do today." (*GT* 293-294, original emphasis)

Keynes draws a sharp distinction between the states of short-term and long-term expectation, which govern production and investment decisions respectively. He would arguably be quite prepared to accept the two EMH assumptions as complements in the case of short-term expectation: in practice, entrepreneurs correct their expectations by trial and error in circumstances which are usually stable over short production periods (EMH-B); and thus for analytical purposes it is acceptable to assume rational short-term expectations (EMH-A):

> "Entrepreneurs have to endeavour to forecast demand. They do not, as a rule, make wildly wrong forecasts of the equilibrium position. But, as the matter is very complex, they do not get it just right; and they endeavour to approximate to the true position by a method of trial and error. Contracting where they find that they are overshooting their market, expanding where the opposite occurs. It corresponds precisely to the higgling of the market by means of which buyers and sellers endeavour to discover the true equilibrium position of supply and demand. … The main point is to distinguish the forces determining the position of equilibrium from the technique of trial and error by means of which the entrepreneur discovers where the position is. … *Ex ante* decisions may be decided by trial and error or by judicious foresight, or (as in fact) by both." (CWXIV, pp. 182-183)

By contrast, "it is of the nature of long-term expectations that they cannot be checked at short intervals in the light of realised results" (*GT* 51). The long-term durable nature of capital assets is precisely the problem; if the expectations upon which the investment was based prove mistaken, it is not possible, either to reverse the investment today, or to go back in time, adjust the original investment decision, and then check the revised results in the present. It is only in a stationary or steady state that adjustments made today might (given stable dynamics) be expected to have the same effect in the future as the same adjustments, made in the past, would have had today. So the convergent feedback mechanism necessary to generate in practice a set of long-term equilibrium prices as the basis of prospective yield is absent in any economy subject to unforeseen change, such as the one we inhabit. It cannot be emphasised enough that it is simply not legitimate to model the real world in terms of long-term equilibrium, because of the irreversible, historical nature of time.

A stationary state with the addition of a stochastic disturbance term and perhaps a deterministic trend (making it a steady state) can be described as an 'ergodic' system (Davidson, 1996). The ergodic hypothesis was originally conceived by Boltzmann in developing the kinetic theory of gases in physical chemistry, to explain the behaviour of macroscopic volumes in terms of the Brownian motion of individual particles. The EMH can be understood as taking markets to generate equilibrium prices in the same way that equilibrium temperatures and pressures are generated by the random collisions of myriads of gas molecules in a closed vessel with a fixed volume. However, the real world is far from stationary, even in a stochastic sense. As Keynes puts it eloquently:

> "The outstanding fact is the extreme precariousness of the basis of knowledge on which our estimates of prospective yield have to be made. Our knowledge of the factors which will govern the yield of an investment some years hence is usually very slight and often negligible. If we speak frankly, we have to admit that our basis of knowledge for

estimating the yield ten years hence of a railway, a copper mine, a textile factory, the goodwill of a patent medicine, an Atlantic liner, a building in the City of London amounts to little and sometimes to nothing; or even five years hence." (*GT* 149)

"By uncertain knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject in this sense to uncertainty…Or, again, the expectation of life is only slightly uncertain. Even the weather is only moderately uncertain. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest 20 years hence, or the obsolescence of a new invention, or the position of private wealth owners in the social system in 1970. About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know." (CWXIV: 113-114)

According to Fama (1970, p. 389), the EMH emerged as a theoretical response to the empirical evidence that stock market prices follow a 'random walk'. A random walk (which also describes Brownian motion) can be expressed as:

$q_{t+1} = q_t + \varepsilon_{t+1}$ where ε is a random disturbance with zero expected value (6)

This must be carefully distinguished from a stationary stochastic process which represents a disturbance about the equilibrium value (note the asterisk):

$$q_{t+1} = q^*_t + \varepsilon_{t+1} \quad (7)$$

If the EMH is to be based on the discovery of the equilibrium position by trial and error (EMH-B), equation (7) alone is the appropriate description, and this can be relevant only where q_t^* is constant or predictable within an ergodic system. The literature to date does not appear to have noted that the consistency of the EMH with the random walk of equation (6) requires perfect foresight of future equilibrium prices, not as a complement or analytical representation of trial and error, but as an independent condition (EMH-A). For if market prices always represent *ex ante* fundamental equilibrium values i.e. $q_t = q_t^*$ under EMH-A, then a random walk may be[5] generated as a result of unpredictable shocks to the endowment, technology and tastes which are the parameters taken to determine future general equilibrium prices. However, the possibility of unpredictable shocks to the *parameters* of the system (and thus to q_t^* rather than q_t) conflicts with the assumption of a stationary (ergodic) state required by EMH-B, where expectations and therefore prices can be wrong in the short term, but the underlying equilibrium price stays put and can be discovered by trial and error. The world required by EMH-B does not generate a random walk, but a stationary stochastic process; the futures markets required, if EMH-A is to be more than an assertion, do not exist. Thus although the EMH purports to explain the empirical evidence of a random walk in prices, it can only do so by asserting EMH-A. The random walk cannot itself be offered as evidence in support of EMH-A.

To summarise the argument so far, it is plausible that in competitive equilibrium the market prices of fixed annuities (q_t) represent *ex ante* their fundamental values (q_t^*). Equally, the fundamental value of any past investment can be determined *ex post* at

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[5] Only 'may be', since the random walk assumes the shocks are randomly distributed and there is no reason to suppose this to be the case in general, especially with technology.

the end of its economic life (q_t^{**}), permitting an historical judgment of the profitability of the initial investment decision. However, the historical nature of time in a world subject to unforeseen change presents insoluble ontological obstacles to the extension of the concept of *ex ante* fundamental value (q_t^*) beyond fixed annuities to financial assets in general. In order to progress beyond this concept, we must consider Keynes's understanding of the nature of long-term expectation.

The state of long-term expectation

Keynes's approach to long-term expectation is informed by his understanding of probability. He treats Classical frequentist probability theory (implicit in the rational expectations hypothesis) as a special case within a branch of philosophical logic that deals with arguments that are doubtful, but neither demonstrably certain nor logically impossible. He understands probability as an argument or logical relation between one set of propositions (the conclusions) and another set (the evidence). Mathematics deals with analytic relations between propositions that must be either true or false. In matters of metaphysics, science and conduct, an argument is considered 'probable' to the extent that it warrants a degree of rational belief. Such a probability relation is objective, in the sense that any rational judge would reach the same conclusion upon the same evidence. Probability is not in general numerical, as in frequentist theory, but arguments can be, and often are, compared. An archetypal case is the verdict reached in a court of law.

Although Keynes treats investors as forming single-valued expectations of prospective yield, these estimates bear a complex relation to the 'bundle of vague and more various possibilities which actually make up their state of expectation when they reach their decisions' (*GT* 24, fn 3), a relation which cannot be reduced to 'actuarial' calculations based on relative frequency. Using Keynes's terminology, I propose that the *ex ante* expectation q_t^* of the *ex post* outcome q_t^{**} is the value of q_t^* which satisfies:

$$\left(q_t^{**} \geq q_t^*\right)\middle|\Omega_t = \left(q_t^{**} \leq q_t^*\right)\middle|\Omega_t \qquad (8)$$

where this expression means that the probability (in Keynes's sense) that the outcome q_t^{**} lies at or above the expectation q_t^* equals the probability that the outcome lies at or below the expectation, given the available evidence Ω_t, including relevant propositions for and against each conclusion[6]. Ω_t is a subset of $\overline{\Omega}$, the complete 'perfect foresight' information set from which q_t^{**} might be known with certainty, i.e. $q_t^{**}\middle|\overline{\Omega} = 1$.

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[6] Equality and inequality do not imply cardinal probability (see Keynes, 1921, pp. 43-44). Glickman (1994) notes the need to distinguish the 'propositions' included in Ω_t from 'events' or raw information which are significant only when interpreted. The term 'information set' must be understood accordingly, and not simply as in standard econometric usage.

The 'expected value' $\mathrm{E}[x]$ of Classical probability theory is in similar fashion given by the centre of gravity of the population relative frequency density function[7] $\varphi(x)$ such that:

$$\mathrm{E}[x] = \int_{-\infty}^{+\infty} x\varphi(x)dx \qquad (9)$$

whence it follows that:

$$\int_{-\infty}^{\mathrm{E}[x]} \varphi(x)dx = \int_{\mathrm{E}[x]}^{+\infty} \varphi(x)dx = 0.5 \qquad (10)$$

Equation (10) is the Classical equivalent of (8), in that q_t^{**} is as likely to fall above $q_t^* = \mathrm{E}_t[q_t^{**}]$ as below it, with the difference that, if we *know* $\varphi(q_t^{**})$, we *know* that, in the limit, half the 'drawings from the urn' will fall on one side and half on the other of the expected value q_t^*. In Keynes's terms, $q_t^* | \varphi(q_t^{**}) = 1$; the *expectation* (although not the actual q_t^{**} itself) is *known* with certainty (as opposed to merely probable in Keynes's sense) as soon as the frequency density function is *known*, since the conclusion follows from the evidence as a matter of strict logical implication: expected value is simply a mathematical transformation of the frequency density function. By contrast, in equation (8), the information set Ω_t does not permit conclusive determination of the expectation q_t^* (let alone, *a fortiori*, the actual value q_t^{**}); or put another way, the two sides of equation (8) do not 'sum' to unity (although strictly these Keynesian probabilities are not in general of the numerical form necessary for addition).

While each side of equation (8) depends on the balance of the evidence for and against each conclusion, the 'weight' of the argument for the expectation q_t^* depends on the relation between the available information Ω_t and the complete information $\overline{\Omega}$. Although no numerical comparison is possible between Ω_t and $\overline{\Omega}$, it is clear that if Ω_t is very scant, little confidence will be placed in the expectation; while if $\Omega_t = \overline{\Omega}$, there will be complete certainty and therefore absolute confidence. Thus the degree of confidence in the expectation q_t^* will depend, although not by a numerical functional relation, upon the weight of the evidence in Ω_t relative to the complete information set $\overline{\Omega}$, which in practice can only be known in retrospect.

From this we can see that the rational expectations hypothesis replaces the assumption of perfect foresight with the only slightly weaker assumption of knowledge of an objective frequency distribution. If we follow EMH-B, that this knowledge can be acquired by discovery in an ergodic system, every addition to the information set Ω_t will improve confidence in the expectation q_t^*, in the sense of reducing its standard error as the sample size increases. In the more general Keynesian case, an addition to the information set Ω_t need not conform to the distribution of previous information in

[7] I use 'relative frequency' to distinguish probability based on frequency from Keynesian probability.

such a well-behaved manner, so that the expectation may fluctuate dramatically. Even if there is considerable weight behind a given expectation, confidence may be shaken by the addition of unexpected bad news; the knowledge that we know so little about the future always haunts us.

In the presence of such fundamental or intractable uncertainty, and in the context of highly liquid investment markets, it is only *rational* to pay more attention to tomorrow's market price q_{t+1}, than to tentative and unreliable expectations of fundamental value q_t^*. At this point we must put aside fundamentals and concern ourselves with the proximate determinants of actual market prices. What really matters, I suggest, is not equation (8) but the rather different:

$$\left(q_{t+1} \geq q_t\left(1+R_{t+1}\right)\right)\middle|\Omega_t = \left(q_{t+1} \leq q_t\left(1+R_{t+1}\right)\right)\middle|\Omega_t \tag{11}$$

which expresses in terms of Keynesian probabilities that tomorrow's price is judged as likely to exceed as to fall short of today's price plus interest; or putting it another way, that the bullish tendency is balanced by the bearish (where these tendencies may exist together in the mind of the same investor or separately among different investors). Information in Ω_t which would not be relevant evidence for the purposes of (8), such as the intentions of other investors, must now dominate consideration of fundamentals. Indeed, the *only* thing that matters (ignoring transactions costs, etc) is the intentions of other investors, so that individual opinions matter only insofar as they contribute to 'average' opinion. If particular investors ('bears') believe the market is over-priced, they should sell today and buy back tomorrow, even if their long-term intention is to hold the asset for its economic life. There may be serious-minded investors in the market whose intentions reflect a model such as equation (2), employing information about current dividends or earnings together with expected growth and interest rates, yet it is still their immediate intentions in the form of arbitrage operations that matter, and not the accuracy of their model, which can only be established long after the event. As Keynes points out in detail in *GT* Chapter 12, the real business of the professional investor must, perforce, be the study of market sentiment, in which the study of fundamental value is at best a minority option. The solution of equation (11) thus provides us with a formal expression of a 'conventional valuation', the price today that balances the bullish and the bearish tendencies in the market as a whole and represents the average opinion or conventional wisdom as to the correct price, given the current information. This price should therefore continue to prevail until there is change in the information - or, of course, in average opinion.

In normal times, quite apart from bubbles, the conventional view of the proper relation between the market price and the current information experiences discontinuous shifts from time to time, perhaps quite frequently. Market prices fluctuate continually, not only as the information set changes, but in line with such changes in conventional valuation. Perhaps the decisive empirical test of the efficient markets hypothesis is the variance bounds test (Shiller, 1981, 2003). The logic of this test is that if prices are a good *ex ante* estimator of fundamental value, the volatility of prices should not exceed the volatility of *ex post* fundamental value. Using US data for 1871-1980, Shiller found that price volatility was at least five times the volatility in fundamental value, rather than less, as the EMH predicts. Much ink has been spilt in an effort to overturn Shiller's claim, motivated presumably by the correct instinct that investors do not normally behave irrationally. Yet if price volatility reflects variation in conventions as well as in the news about technology, preferences and endowment, Shiller's result is fully to be expected, since conventions may change frequently without a descent into irrationality. Furthermore, equity prices will follow a random walk, if news is unpredictable and *a fortiori* if changes in conventions are also unpredictable.

Conclusions

The concept of *ex ante* fundamental value reaches into every corner of orthodox thinking about financial markets, yet neglects the self-evident facts that time is irreversible and the world is subject to unforeseen change, which cannot be reduced to a frequency distribution. Once it is admitted that *ex ante* fundamental value can have no operational meaning beyond the case of fixed annuities, the concept of rationality must also be reassessed: it has been too easy for orthodoxy to create a false division between rational (meaning objectively optimal) and irrational behaviour. On the contrary, we have seen that conventional valuation is an entirely rational Socratic response to the knowledge of our lack of knowledge of the future, even though conventions must be based on the psychology of the market, and only indirectly, at best, on investors' models of fundamental value. This is by no means to deny the possibility of irrational behaviour (e.g. during bubbles), but the alternative perspective substantially affects the manner in which we approach the empirical evidence.

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