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Critical current enhancement by Lorentz force reduction in superconductor–ferromagnet nanocomposites

M G Blamire, R B Dinner, S C Wimbush and J L MacManus-Driscoll

Department of Materials Science, University of Cambridge, Pembroke Street, Cambridge CB2 3QZ, UK

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Abstract
Ferromagnetic pinning centres in superconductors form much deeper potential wells than equivalent insulating or metallic non-superconducting inclusions. However, the resultant pinning forces arising from magnetic inclusions are low because the magnetic interaction takes place over the length scale of the magnetic penetration depth which is large in technological superconductors. Nonetheless, we show that a magnetic inclusion can also reduce the Lorentz force on a vortex, yielding a substantially enhanced critical current density for a given pinning force. We calculate this enhancement for a single vortex pinned by a paramagnetic cylinder as well as a vortex lattice interacting with magnetic inclusions, and find that the inclusion of ferromagnetic particles or rods offers a practical means of enhancing the critical currents in oxide high temperature superconductors.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

There has been considerable recent interest in the interplay of magnetism and superconductivity [1–3]. While much of this has centred on the fundamental electronic interactions and has been enabled by considerable advances in the nanofabrication techniques necessary to study the very short-range effects [4–6], there is also considerable practical interest in magnetic interactions between the systems to enhance critical current densities of superconductors.

In any type II superconductor, the Lorentz force $\mathbf{J} \times \mathbf{B}$ on the vortex lattice is opposed by interactions between the vortices and pinning centres. Once the maximum pinning force is exceeded at the critical current density (J_c), vortex flow generates an electric field and hence a loss of practical superconductivity [7]. The concept of 'magnetic pinning' has been investigated over a number of years [8, 9], and has been advanced in recent years as a possible means to enhance the J_c of practical conductors based on high temperature superconductors (HTS) [10–12]. This form of pinning is expected to be achieved most effectively by incorporating ferromagnetic (FM) nanoparticles or nanorods within the superconducting (S) matrix. An apparently straightforward argument based on the relative energies of magnetic pinning compared with those of conventional core pinning suggest that this is a potentially fruitful avenue to explore despite the technological difficulty of fabricating the sample geometries required.

In a previous publication [13] we showed that with very high volume fractions (∼30%) of small FM particles in low temperature superconductors, a new pinning regime based on magnetic hysteresis losses was observable. Although of relevance to the emerging study of ferromagnetic superconductors, the actual enhancement of J_c over a plain superconducting film was small. In this paper we consider an alternative mechanism in which FM inclusions enhance J_c, by reducing the Lorentz force acting on vortices.

Conventional core pinning of vortices is due to the savings in condensation energy associated with the vortex core which arise when the core is situated within a non-superconducting

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inclusion. The energy per unit length of vortex associated with core pinning is then given by:

$$\varepsilon_{\mathrm{core}} \approx \mu_0 H_{\mathrm{c}}^2 \xi^2 \approx \frac{\Phi_0^2}{\lambda^2} \tag{1}$$

where Φ_0 is the flux quantum, H_{c} is the thermodynamic critical field, ξ is the coherence length and λ is the magnetic penetration depth. In contrast, 'magnetic pinning' is associated with the Zeeman energy per unit length of particles magnetized by vortices and so can be of the order

$$\varepsilon_{\mathrm{mag}} = \frac{\mu_0}{2} \int_{\mathrm{area}} MH \approx \frac{1}{2} M \Phi_0 \tag{2}$$

where M is the magnetization of the ferromagnet and H is the local magnetic field from the vortex. If reasonable values for the various parameters for HTS and strong FMs are substituted into equations (1) and (2) then the energies associated with magnetic pinning are several orders of magnitude larger than those for core pinning. However, the important factor in determining J_{c} is actually the pinning force, i.e. the maximum gradient of the pinning potential, approximately equal to the depth of the pinning potential divided by its characteristic length scale: ξ for core pinning and λ for magnetic pinning. In high $\kappa(=\lambda/\xi)$ materials such as HTS this means that the potential gain in pinning force for magnetic inclusions appears to be somewhat limited.

In this paper we introduce a key factor which appears to have been overlooked in previous discussions of magnetic pinning: that of the modification of the Lorentz force on vortices associated with ferromagnetic pinning centres. The primary difference between the calculations presented in this paper and those which have been previously published [14, 15, 12] is that here we explicitly include the field dependence of the magnetization of the ferromagnetic inclusions rather than considering their moments to be essentially constant. Practically, this requires that soft magnetic materials need to be used if such pinning is to be effective.

The structure of the paper is as follows: we will first consider the interaction of an isolated vortex with a parallel magnetic rod which enables the low-field behaviour to be analysed. We will then consider the case at higher fields which is more appropriate for applications. Finally, the practical consequences and the requirements for significant critical current enhancement will be addressed.

2. Flux transfer from a single vortex to a magnetic inclusion

For simplicity, our analysis assumes the use of an ideally soft ferromagnetic material in the form of a cylinder parallel to the applied field and vortices within the superconductor and that the system is effectively infinite in the field direction. Later we will consider the effect of more realistic materials and thin film geometries.

A single vortex is placed at a distance r from a parallel magnetic cylinder of radius R as shown in figure 1. The

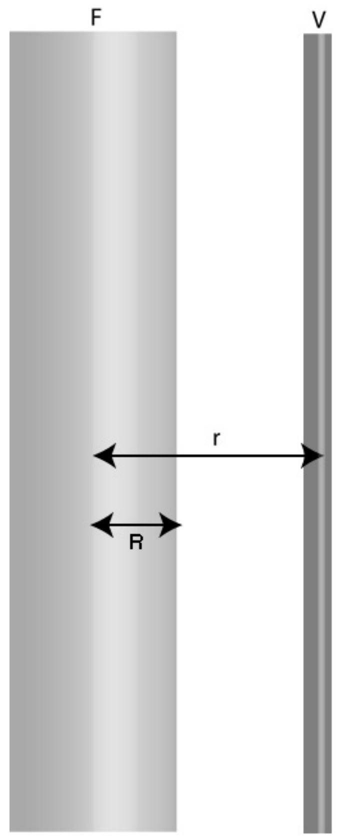

Figure 1. An isolated vortex (V) parallel to a magnetic rod (F) showing the length scales of the interaction.

field H at the cylinder surface arising from the vortex screening currents must be continuous across the S/FM interface, so the local magnetization M of the cylinder must be $M(H)$. We assume that the ferromagnet is ideally soft (i.e. superparamagnetic) so that $M = (1+\chi)H$ and $\chi \gg 1$.

In the high κ limit, the local field at a distance $r > \xi$ from a straight vortex is given by

$$H(r) = \frac{\Phi'}{2\pi\mu_0\lambda^2} K_0\left(\frac{r}{\lambda}\right) \tag{3}$$

where the total magnetic flux Φ' is equal to the flux quantum Φ_0 for an isolated vortex (but is here allowed to vary) and K_0 is a modified Bessel or MacDonald function. In the limit $\lambda > r \gg R$ we can assume that the field is constant over the magnetic cylinder, and so the flux due to the magnetization of the cylinder is given by

$$\phi = \mu_0(\chi+1)\pi R^2 \left(\frac{\Phi'}{2\pi\mu_0\lambda^2}\right) K_0\left(\frac{r}{\lambda}\right). \tag{4}$$

However, if we choose a flux integration path well outside the regions of the screening currents and the magnetic cylinder so that the flux is equal to the fluxoid, then the total flux enclosed must remain quantized such that

$$\Phi_0 = \Phi' + \phi$$

and so

$$\Phi_0 = \Phi' + \frac{(\chi+1)R^2\Phi'}{2\lambda^2}K_0\left(\frac{r}{\lambda}\right) \quad (5)$$

for self-consistency. Thus the flux associated with the vortex, Φ', must decrease as the vortex approaches the ferromagnetic cylinder to compensate for the additional flux within the cylinder:

$$\Phi' = \frac{\Phi_0}{1+\frac{(\chi+1)R^2}{2\lambda^2}K_0\left(\frac{r}{\lambda}\right)}. \quad (6)$$

This variation of this flux with r is illustrated in figure 2(a). The effect of a magnetic inclusion, $\chi = 100$, is contrasted with a non-magnetic inclusion, $\chi = 0$.

This is the first significant result of this paper and, in itself, should not be unexpected. Even in the non-magnetic case of a vortex approaching a vacuum inclusion, flux is lost from the vortex as it approaches the interface: for a cylindrical defect in a bulk superconductor this can be calculated analytically [16] and for thin films approximate solutions exist [17]. Here however, the large susceptibility χ enhances the effect. A related calculation has been performed by Genkin *et al* [18]; however the system modelled in that paper consists of a S/FM bilayer, so the quantization condition differs from the cases considered here.

3. Flux loss from the vortex: consequences for the pinning potential

The transfer of magnetic flux from a vortex to a magnetic inclusion results in two savings in free energy per length that contribute to a magnetic pinning potential. Firstly, the Zeeman energy per unit length of the cylinder ($-\pi R^2 MH/2$) is negative. Secondly, the line energy of the magnetic field of the vortex, $(\Phi'(r)/4\pi\lambda^2)^2 \ln\kappa$ reduces as $(\Phi'/\Phi_0)^2$. Both of these energies vary over a length scale of λ, and so create the wider potential well illustrated in figure 2(b) for a technologically relevant superconductor where $\kappa \gg 1$.

On top of this magnetic pinning potential, we have added the narrow potential well that arises from core pinning associated with the absence of the superconductivity in the ferromagnetic cylinder (equation (1)) which varies over ξ as the vortex core crosses into the non-superconducting inclusion, (and has a flat bottom of width $2R$). Taking the gradient of this combined potential (figure 2(c)), we see that the maximum pinning force is largely unchanged between non-magnetic and magnetic inclusions because it is dominated by core pinning which is the same for magnetic and non-magnetic inclusions. If this were the whole story, magnetic inclusions would not enhance the critical current any more than their non-magnetic equivalents. In the next section we show that it is actually the reduced Lorentz force on a vortex in the vicinity of a ferromagnetic cylinder which potentially enhances the critical current.

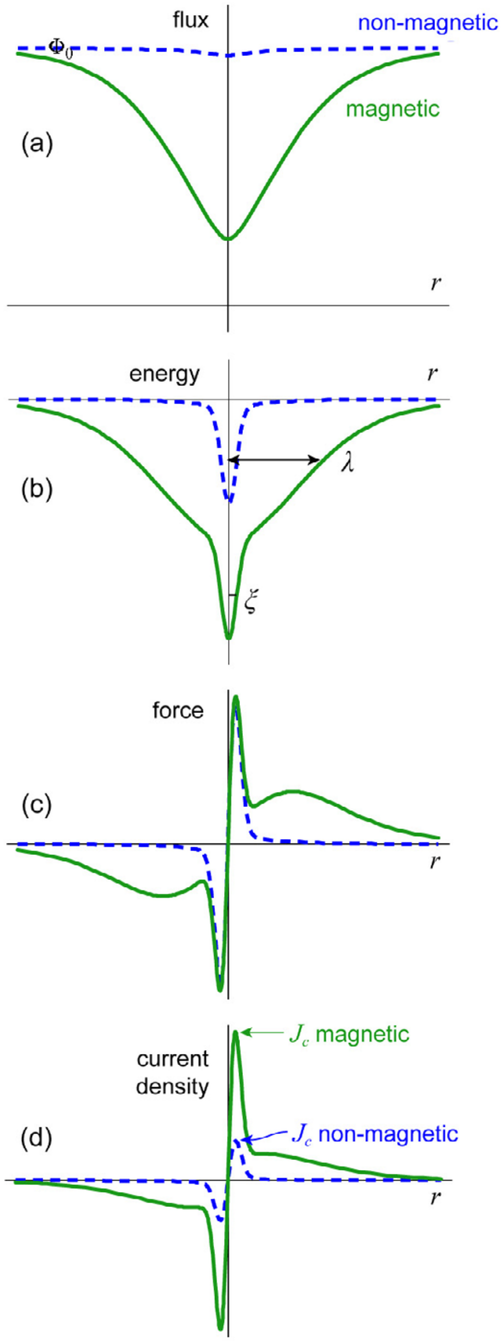

Figure 2. The dependence of quantities on r, the distance of a vortex relative to a parallel cylindrical pinning site. Calculations for magnetic ($\chi = 100$) and non-magnetic cylinders are shown: (a) the flux remaining in the vortex is reduced as it approaches the cylinder in accordance with equation (6); (b) the pinning potential consists of core pinning on the length scale of ξ and reduced magnetic energy on the length scale of λ, following equations (1) and (6). (c) The pinning force derived as the slope of the above potential. (d) The current density necessary to balance the pinning force is greatly enhanced in the magnetic case (equation (7)).

4. Reduced Lorentz force

As discussed in detail by Nozières and Vinen [19] the Lorentz force within the superconductor is more correctly viewed as a Magnus force arising from the interaction of the externally-applied transport current with the rotating screening currents of the vortex. The reduced flux of the vortex in the magnetic case necessarily leads to reduced screening currents and hence a lower net force on each vortex. Although electromagnetism dictates that the full Lorentz force is experienced by the vortex-magnet system, most of this force is applied to the immovable magnet, rather than the vortex. Since flux motion takes place under the Lorentz force per unit length on the vortex:

$$f_{\rm L} = J\Phi' \tag{7}$$

the Lorentz force on the vortex is therefore reduced in proportion to its flux as given by equation (6). For example, we plot, as a function of r, the current necessary to balance the pinning force (figure 2(d)), $J(r) = F_{\rm pin}(r)/\Phi'(r)$. The critical current is the maximum of this plot and it shows a significant enhancement of critical current density ($J_{\rm c}$) in the magnetic case. This result applies equally to core pinning associated with intrinsic defects within the superconductor such that $J_{\rm c}$ is enhanced in proportion to the reduction of Φ'.

5. From single vortex to vortex lattice

In a macroscopic system in which a lattice of multiple vortices is stabilized against flux flow both by core pinning and by the presence of multiple ferromagnetic cylinders or particles the enhancement in the critical current density is given by evaluating the total Lorentz force on the vortex lattice. We can generalize equation (7) to express this as:

$$\frac{J_{\rm c}'}{J_{\rm c}} = (1-f)\frac{\bar{B}}{B'} \tag{8}$$

where $\bar{B}$ is the mean flux density within the whole system, B' is the residual flux density in the superconductor in the presence of the ferromagnetic inclusions and f is the volume fraction of the ferromagnet (and hence areal fraction so that the current-carrying cross-sectional area of the conductor is reduced). B' is a function of f, but the form of the dependence, and its link to the applied field, is dependent on the geometry of the sample, the degree of ordering within the ferromagnetic inclusions and the shape of the inclusions (for example whether they are rods or spherical nanoparticles).

To arrive at a simple expression which can be compared with experiment we confine the analysis to a field range in which $\bar{B} \sim \mu_0 H$. For thin films in perpendicular field, this is actually most of the field range and so is not particularly restrictive. If we also assume that the ferromagnetic inclusions are rods parallel to the field with radius R which is substantially less than the film thickness, then we can impose an overall continuity of flux density to provide the link between the internal $H' = B'/\mu_0$ and external magnetic fields H so that:

$$\bar{B} = \mu_0 H = \mu_0\left(H' + M(H')\right) f + \mu_0 H'(1-f) \tag{9}$$

where the first term of the equation relates to the flux within the inclusions and the second term is the residual flux within the superconducting matrix. Then

$$\bar{B} = \mu_0 M\left(B'/\mu_0\right) f + B'. \tag{10}$$

The two equations (8) and (10) therefore link the magnetic properties of the superconductor with ferromagnetic rods to the critical current enhancement. It can be seen that for high fields well above the saturation field of the ferromagnetic rods, the critical current enhancement is negligible since there is then relatively little flux extracted from the vortex system.

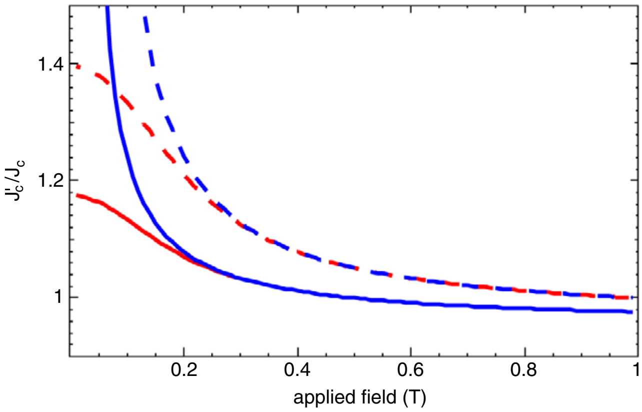

Figure 3. The critical current enhancement versus applied field for a type II superconductor containing a 5% volume fraction of ferromagnetic nanoparticles. The different curves correspond to a saturation magnetization ($\mu_0 M_{\rm s}$) of 0.5 T (solid lines) and 1 T (dashed lines) and a zero-field relative susceptibility ($\chi/\mu_0 M_{\rm s}$) of 8 T^{-1} (red lines) and 80 T^{-1} (blue lines (divergent at low fields)).

6. Practical consequences of FM pinning on $J_{\rm c}$: enhancement in oxide superconductors

We can now estimate the $J_{\rm c}$ enhancement assuming realistic parameters for ferromagnetic inclusions in HTS. We consider firstly, the isolated vortex limit of section 4 which requires that the vortex separation is much greater than λ. We assume that the inclusions are rods of soft oxide (for example various of the ferrites) and hence $\chi \gg 10^2$ should be possible [20, 21]. It has been widely believed for some time that core pinning, probably induced by a high density of intrinsic defects, provides the strong intrinsic pinning in HTS films [22]. For an inclusion diameter of 30 nm, equations (6) and (7) suggest a pinning enhancement of greater than a factor of a hundred at very low fields. In fact, however, zero-field $J_{\rm c}$ values for a typical HTS such as $YBa_2Cu_3O_{7-x}$ (YBCO) are within a factor of 10 of the depairing current density, and the absolute enhancement of $J_{\rm c}$ must be limited to this value.

For YBCO, the in-plane penetration depth λ_{ab} is 230 nm at 77 K so this isolated vortex limit actually corresponds to a field range below 50 mT: it is therefore much more realistic to use equations (8) and (10) to estimate the $J_{\rm c}$ enhancement. Figure 3 shows the predicted enhancement in the critical current density for a low volume fraction as a function of applied field assuming two different values of the saturation magnetization and zero-field susceptibility. From the figure it is clear that, although high susceptibility can give rise to very large enhancements close to zero field, the saturation magnetization, $M_{\rm s}$, is likely to be of more significance than the susceptibility for enhancing $J_{\rm c}$ in practical fields, since even the self-field of an HTS conductor at $J_{\rm c}$ is usually more than 100 mT. Typical values of $\mu_0 M_{\rm s}$ for nanoparticles of soft

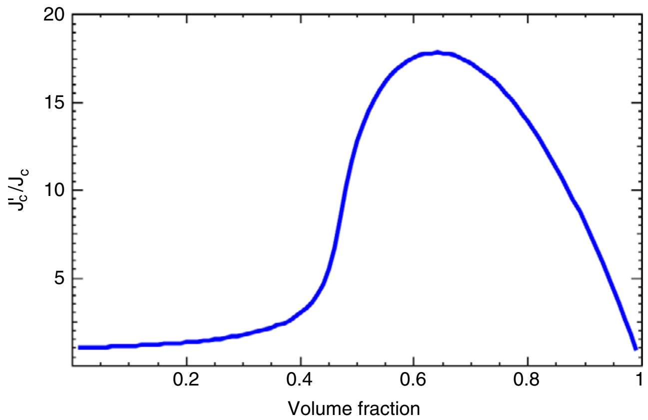

Figure 4. The critical current enhancement versus volume fraction of ferromagnetic rods at an applied field of 0.5 T. The ferromagnet has a saturation magnetization ($\mu_0 M$) of 1 T and a relative susceptibility ($\chi/\mu_0 M_s$) of 80 T^{-1}.

ferromagnetic oxides are 0.5–1 T [20, 21] and so substantial J_c enhancements will be limited to fields below this range.

Figure 4 shows the predicted enhancement in the critical current density as a function of volume fraction f for a fixed field which is half the saturation magnetization. It can be seen that very large enhancements are possible for large f, but practical considerations in terms of improving the properties of high temperature superconductors, such as maintaining epitaxial growth, probably require that f is substantially lower than 0.5.

The general case of randomly dispersed particles is more difficult to analyse, because flux continuity requires that the enhanced flux density within a magnetic nanoparticle re-penetrates the superconductor parallel to the vortex direction. This must locally modify the internal field and lead both to distortion of the vortex lattice and the conventional magnetic pinning. This is a problem considered by Kruchinin *et al* [23], but they considered the problem from the point of view of the magnetostatic coupling between particles and did not explicitly consider the properties of the vortex lattice. In general however, the local reduction in Lorentz force on a vortex passing near a particle still appears to apply, and so a significant J_c enhancement at low fields is still expected.

In our analysis we have assumed that the ferromagnetic particles are fully reversible (superparamagnetic). It is indeed possible to control the properties of isolated ferromagnetic oxide nanoparticles so that the superparamagnetic blocking temperature is below 50 K [21]; however, the size reduction required is accompanied by a loss of magnetization and so in practice it may be necessary to use ferromagnetic inclusions which have a hysteretic field response. At first sight it would appear that this would necessarily result in a hysteretic $J_c(H)$ response at low field. However, the important field is not the applied field but the local field of the vortices: provided the local field is larger than the coercive field H_c, and so is sufficiently large to reverse the field in the rods or particles in the vicinity of the vortex, then any hysteresis in $J_c(H)$ will be small. Previous experiments [8, 9] demonstrate an enhanced coercivity in the magnetization of Hg/Fe nanocomposites consistent with enhanced pinning, albeit at low fields. The opposite limit in which H_c is much larger than the vortex field will give a highly asymmetric response of J_c to applied field as discussed by Lyuksyutov and Naugle [24]. It would appear that no experiments have investigated the soft FM regime, although the work of Lange *et al* [5] in which the stray field from magnetic dots on the surface of a superconductor results in an asymmetric critical temperature ($T_c(H)$) demonstrates that magnetic particles with the correct properties can be realized.

7. Conclusions

We have shown that paramagnetic or ferromagnetic pinning centres have the potential to create a significant enhancement of the critical current density of a type II superconductor and that the primary mechanism at low fields is the effective reduction of the flux within vortices which reduces the Lorentz force on the vortex lattice. The enhancement is likely to be limited to fields below 1 T by the saturation magnetization of the magnetic inclusions, but for low-field applications such as power transmission and motors, the pinning enhancements predicted here should apply. In predicting the critical current density we have assumed that the core pinning in the superconductor is unaffected by the insertion of ferromagnetic pins. In reality, ferromagnetic pins of the correct structure and geometry should further promote core pinning in the same way that non-magnetic inclusions do and so our results may underestimate the critical current enhancement possible.

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